



Christ Church
Grammar School

2019
TEST 5

MATHEMATICS SPECIALIST Year 12

Section One: Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	15 minutes
Marks available:	13 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

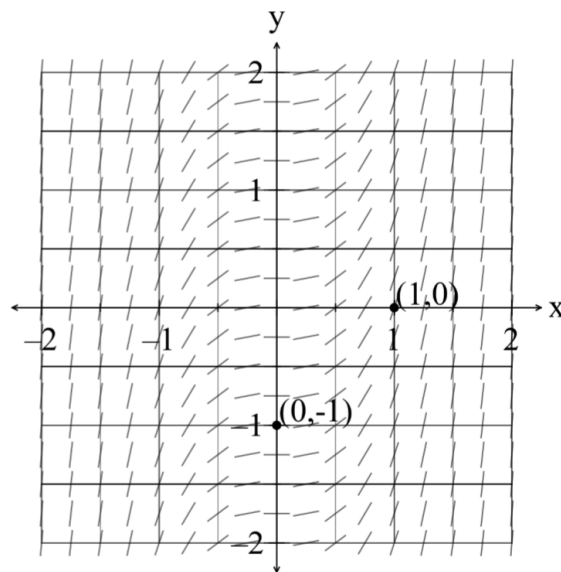
Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1

(5 marks)

A first order differential equation has a slope field as shown in the diagram below.



- (a) Determine a general differential equation that is represented by this slope field. (2 marks)
- (b) Find the unique solution to the differential equation in part (a) given that the points $(1, 0)$ and $(0, -1)$ belong to the curve. (3 marks)

Question 2**(8 marks)**

If, at any time, a particle moves along a straight line with acceleration, a , cm/s², velocity, v , cm/s and displacement, x , cm from a fixed point O, the relationship between these variables is given by

$$a = \frac{dv}{dt} \text{ and } v = \frac{dx}{dt}$$

For the case when $a(t) = e^{2x}$ with initial conditions $x(0) = 1$ cm and $v(0) = e$ cm/s, determine

- (a) the exact velocity of the particle when $x(t) = 5$ cm. **(4 marks)**

Question 2 continued

- (b) the exact time when the particle will have a displacement of 10 cm to the right of the origin. (4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____



MATHEMATICS SPECIALIST Year 12

Section Two:

Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time for this section:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

Question 3**(7 marks)**

A particle moves along the $x - axis$ so that its displacement x , in metres, from the origin at any time t , in seconds, is given by:

$$x = 5 + 2\cos(4t)$$

- (a) Show that the particle demonstrates simple harmonic motion about some point P on the $x - axis$. (3 marks)

- (b) Find the point(s) on its path of motion where its speed is 4 ms^{-1} . (3 marks)

- (c) Determine the acceleration of the particle when it is located 1 m to the left of P . (1 mark)

Question 4**(7 marks)**

The population $N(t)$ of numbats in a fauna reserve at time t years is modelled by the differential equation, $\frac{dN}{dt} = 0.000002N(500 - N)$ where $N(0) = 250$.

- (a) Use integration to show that $N(t) = \frac{A}{1+Be^{-kt}}$, stating the values of A, B and k .
(6 marks)

Question 4 continued

- (b) If current conditions prevail, then what is the expected long-term population of numbats in the reserve? (1 mark)

Question 5**(8 marks)**

The shape formed when the area in the first quadrant, which is enclosed between the x - axis, the y - axis, $y = x^2 - 4$ and $y = h$ ($h > 0$), is rotated about the y - axis, is the same as the interior of a flower vase.

- (a) With one centimetre to one unit on each axis determine the value of h , to the nearest centimetre, if the capacity of the vase is 2.5 L. (4 marks)

Question 5 continued

- (b) Water is poured into this vase at a constant rate of $5 \text{ cm}^3/\text{s}$. Determine the rate at which the water level is rising when the depth of water in the vase is 6 cm.
(4 marks)

Question 6**(8 marks)**

An internet service provider plans to sample the volume of content downloaded per day by customers subscribing to their ADSL20 plan. From recent research, the company knew that the volume of downloads per customer was normally distributed with standard deviation 1.4 GB.

- (a) Determine how large a sample the company should take in order to be 90% confident that the mean volume of downloads per customer calculated from their sample is within 0.25 GB of the true population mean. (2 marks)
- (b) A random selection of 25 subscribers was made and the total volume downloaded by these customers over a 24 hour period was 120 GB. Calculate a 95% confidence interval for the mean volume of content downloaded per day by a customer. (4 marks)
- (c) If the company repeated the random sampling process and subsequent 95% confidence interval calculations from part (b) a total of 40 times, then how many of the intervals calculated would you expect to contain the true population mean? Justify your answer. (2 marks)

End of Questions

Additional working space

Question number: _____

Additional working space

Question number: _____